Ghost Seats in the Basque Parliament*

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May 29, 2015

Abstract

In elections voters generally have four options: to abstain, to cast a blank vote, to cast a null vote, or to vote for a candidate or party. This last option is a positive expression of support, while the other three options reflect lack of interest, or dissatisfaction with the parties or the political system. However only votes for parties or candidates are taken into account in the apportionment method. In particular the number of seats allocated to parties remains constant even if the number of non votes (i.e. blank votes, null votes and abstentions) is very large. This paper proposes that non votes be treated as a party in the apportionment method and that the corresponding seats should be left vacant. These vacant seats are referred to as "ghost seats". How this would affect decision-making is quantified in terms of power indices. We apply this proposal to a case study: the regional parliament of the Basque Autonomous Community (Spain) from 1980 till 2012.

1 Introduction

Electoral turnout has decreased in most democratic countries in the past few decades. Since 1988 the average turnout in EU Members States has been around 78% compared to almost 84% before 1987. The average hides significant differences between countries. In Belgium, Luxembourg and Italy, around 90% of the electorate usually vote, while in Ireland, France and Portugal turnout is less than 75% (IDEA, 2004). The 2014 elections to the European Parliament exhibit the lowest turnout on record: only 42.54%

*The support of the Spanish Ministerio de Economía y Competitividad under project ECO2012-31346, and of the Departamento de Educación, Política Lingüística y Cultura from the Basque Government (Research Group IT568-13) is gratefully acknowledged.

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of Europeans voted, and turnout did not even reach 20% in the Czech Republic or Slovakia. As put by Muxel (2009) the party of abstainers would be the biggest political group in the European Parliament.

Part of this abstention may be circumstantial (illness, absence from home, registration problems) but the rest is voluntary. Blondel et al. (1997) propose the following explanations for voluntary abstention in European Parliament elections: lack of interest, lack of knowledge, distrust or dissatisfaction with politics and politicians, distrust or dissatisfaction with the electoral process. According to Delwit (2013) the growth of abstention is a sign of indifference toward or distrust of politicians and politics.

Among "non votes" (i.e. abstentions, blank votes and null votes) abstention has attracted the lion's share of attention in the literature. See however Power and Roberts (1995), Power and Garand (2007), Uggla (2008) and Troumpounis (2011). One reason for the scant attention paid to the other two forms of expression[1] is that they are notoriously difficult to interpret. As put by Damore et al. (2012), the number of "non votes" is part of the signal that the election sends to the political system, but the signal is not clear. Non votes may arise from motivations varying from alienation to boredom or confusion. Another reason may be that null and blank votes represent a very low percentage of the electorate. Nevertheless the phenomenon has become increasingly common, as has voting for extra-parliamentary parties (Uggla, 2008). For instance in France the percentage of blank votes increased from 2.5% in 1990 to 5% in 2000 (Zulfikarpasic, 2001). It even reached between 9 and 20% in districts where the competition in the second round was between two candidates from the same side of the electoral axis (two right-wing candidates or two left-wing candidates). Power and Garand (2007) point out that these percentages may be not negligible in some countries, especially in Latin America (with an average of 11% of invalid votes, with a range from 2-3% in some countries to 20-30% in others).

A blank vote is usually intentional, although it may sometimes be a form of hidden abstention. It may express dissatisfaction with politics or politicians or inability to choose between candidates (Zulfikarpasic, 2001). Teixidor (2012) distinguishes between null votes due to inexperience or error and those which are intentionally null (to show political disagreement). The respective proportions of intentional and unintentional null votes cannot be evaluated. The following examples illustrate clear signals of disagreement or dissatisfaction sent by electors through non votes. In 1928 in Antwerp (Belgium) the by-election to replace a deceased liberal member of Parliament was transformed into a plebiscite on the nation's language policy. Some parties called for a boycott of the election and as a result 31.3% of the ballots were spoiled[2] (van Goethem, 2010, p 153-154). In the 1969 French presidential election the defeated communist candidate in the first round called on his supporters to abstain in the second round. As a result abstention rose from 21.8% in the first round to 30.9%, and

[1] As pointed out by Teixidor (2012) some countries do not recognize the blank vote option. In these cases blank votes are considered as null votes.

[2] We thank Luc Bovens for drawing our attention to this example.

in parallel blank votes increased from 1% to 4.5% (Zulfikarpasic, 2001). In the 2001 Argentine elections 20% of the votes cast were either blank or null with the clear objective of casting an "angry vote" (Uggla, 2008). In the 2009 elections to the Basque Parliament the illegalized party Batasuna called on its supporters to cast a null vote and 5.7% of the electorate did so. Note that disagreement or dissatisfaction can also be hidden behind vote for an alternative candidate. In the second round of the 2002 French presidential elections, the right wing candidate Chirac was chosen by 82.21% of the voters. Many of the votes cast in favor of Chirac were votes against his opponent, the extreme right candidate Le Pen.

The "None of the Above" (NOTA) option has the advantage of offering a non ambiguous means to indicate dissatisfaction. Voters bear the cost of electoral participation in order to send this signal. This option is however used in few countries. A well-known example is the state of Nevada where it was chosen by an average of 10.98% of the electorate during the 1976-2010 period (Damore et al., 2012). The NOTA option was introduced in Russia in the 1993 electoral reform and suppressed in 2006 in spite of its success. For instance in 2003 this option was the second most voted "party" with nearly 13% of the votes. In some constituencies it achieved more votes than the largest party (Mc Allister and White, 2008). In 2013 the NOTA option was introduced in India by the Supreme Court. So far (that is, in the first general elections held since, in 2014) it has obtained a modest 1.1% of the votes (Diwakar, 2015).

In France there has been some movement towards having the blank option recognized. Since 2014 blank and null votes are counted separately even though both options are considered as invalid and do not count in the tally.[3] Zulfikarpasic (2001) reports that in 1992 a party whose objective was to represent blank votes obtained one seat in regional elections in Brittany (and got 5% of the votes). This is also the case in Spain where there is a party ("Blank Seats") whose program is to reform the electoral law so that blank votes are transformed into vacant seats in the Parliament. Meanwhile their elected candidates will decline to sit in parliament. They achieved a total of four seats in the 2011 local elections in Catalonia[4] (two out of seven in Foixá, one out of thirteen in Gironella, and one of thirteen in Santa Maria de Palautordera).

The objective of this paper is to propose some way of evaluating the potential effect of such a proposal. More generally we set out to measure the impact that each category of non votes could have in parliaments if it counted as a party in the apportionment method. We apply the apportionment method to the list of competing parties, to which we add a fictitious party whose number of votes is equal to the number of blank votes, to the number of abstentions, or to the number of null votes. Three alternative distributions of seats are obtained, each of which is compared to the actual distribution (that is, the one obtained when the apportionment method is exclusively applied to the competing parties). In particular we quantify the number of "ghost seats", i.e.

[3] By contrast in other countries blank votes are considered as valid and count in computing the threshold for a party to obtain representation while null votes are not.

[4] See the website of the Spanish Ministry of the Interior: www.infoelectoral.mir.es

the seats that would correspond to the votes of the additional fictitious party. We then analyze how the three alternative apportionments would affect decision-making in parliament (i.e. the weighted majorities among the parties if the "ghost seats" are left vacant).

There is a vast body of literature on assessing decision-making in parliaments. Many power indices have been proposed to quantify the parties' ability to influence outcomes. The most popular include the Shapley-Shubik (1954), Banzhaf (1965), Rae (1969), and Coleman (1971, 1986) indices. Here we follow the probabilistic approach proposed by Laruelle and Valenciano (2005, 2008) and measure the respective probabilities of acceptance of a proposal and probabilities of obtaining the preferred outcome. As the objective is a normative comparison of the decision-making attributable to the different apportionments, we leave out of our model elements that should be incorporated, if the objective were to give an accurate descriptive account of the parliament. Thus, we ignore the information concerning party location in the political space as inconsistent with the normative approach. The only relevant information is the distribution of seats.

The extent of the effect on a parliament needs to be evaluated via a case study. In this paper we focus on the regional parliament of the Basque Autonomous Community (Spain) from 1980 till 2012. The apportionment rule is the D'Hondt method with a threshold. In each of the three districts (one per province), one third of the seats is allocated. For all years we use the same apportionment method (with the same threshold) but add the blank votes, the null votes or the abstentions to the list of competing parties. We then compare the three resulting apportionments with the actual one. The results show that (except in 2009) neither blank votes nor null votes would affect the distribution of seats. By contrast the potential effect of abstention is very substantial. More than 20% of the seats in the parliament would be ghost seats. The presence of ghost seats has the effect of increasing the quota in relative terms. This decreases the probability a proposal being approved. It also decreases a party's likelihood of getting a proposal adopted when it favors it. Nevertheless the probability of having a proposal rejected when the party is against it increases. Moreover in each legislature there is at least one party which can act as a vetoer. The overall effect is that a party's probability of obtaining its preferred outcome generally decreases. We expect the potential effect of the abstention to be just as large in all countries that exhibit similar rates of abstention (33.6% of the electorate).

The limits of the exercise must be kept in mind. First it is difficult to predict what voters would really do. Throughout the exercise we assume that the modification of the apportionment method would not alter the numbers of blank votes, null votes or abstentions. We also assume that all voters who vote for a given party would continue voting for it. Second the behavior of the parties is another unknown: It is clear that leaving seats vacant the seats would be a clear incentive for politicians to pay more attention to non voters. Third, the objective of the exercise is normative: the probabilities obtained cannot capture the similarities and differences between parties in political space and thus do not provide a descriptive analysis.

The rest of the paper is organized as follows: Section 2 details how the effect of abstention, null and blank votes is quantified. Section 3 analyzes the case of the Basque Parliament. Section 4 concludes with some remarks. The Appendix contains a description of the Basque party system.

2 Quantifying the effect of non votes

In electoral year t the electorate (denoted e_t) can be decomposed into the number of abstainers (which we denote by a_t), the number of blank votes (which we denote by b_t), the number of null votes (which we denote by c_t) and the votes actually cast for the different parties. Let m_t denote the number of competing parties, and M_t their set. If party $i \in M_t$ has obtained $v_t(i)$ votes, then

$$e_t = a_t + b_t + c_t + \sum_{i \in M_t} v_t(i).$$

An apportionment method allocates the k_t seats in the parliament (or the k_t seats in the district) among the parties on the basis of the votes for the different parties. Formally it is a function F that associates with any vector of votes $\vec{V}_t^0 = (v_t(1), ..., v_t(m_t))$ a vector of seats $\vec{Y}_t^0 = (y_t^0(1), ..., y_t^0(m_t))$: $\vec{Y}_t^0 = F(\vec{V}_t^0)$ with

$$y_t^0(i) \geq 0 \text{ for any } i \in M_t, \text{ and } \sum_{i \in M_t} y_t^0(i) = k_t.$$

The apportionment method satisfies the property of monotonicity: a party with more votes than another should not have fewer seats

$$\text{if } v_t(i) < v_t(j) \text{ then } y_t^0(i) \leq y_t^0(j). \tag{1}$$

For an analysis of apportionment methods, see Balinski and Young (1982).

Let N_t^0 be the set of parties with representation in the parliament in apportionment $\vec{Y}_t^0$. That is, $N_t^0 = \{i \in M_t \mid y_t^0(i) \neq 0\}$ is the set of parties with a non null number of seats in apportionment $\vec{Y}_t^0$. The number of parties with representation is denoted by n_t^0 and the corresponding distribution of seats in the Parliament is given by $\vec{W}_t^0 = (w_t^0(1), ..., w_t^0(n_t^0))$ with $w_t^0(i) = y_t^0(i)$ for any $i \in N_t^0$.

The apportionment method only takes into account the votes actually cast for parties. That is, it is as if those who cast non votes were not part of the electorate. Why are these votes not included in the apportionment? How many seats would correspond to abstention (A)? blank votes (B)? and null votes (C)? How would the distribution of seats among the parties be affected?

For $Z = A, B, C$, let $\vec{V}_t^Z$ denote the vectors of votes for the different parties if a fictitious party is added whose number of votes is equal to the votes received by category Z of non votes. That is,

$$\vec{V}_t^Z = (v_t(1), ..., v_t(m_t), z_t) \text{ with } z_t = a_t \text{ if } Z = A,\ z_t = b_t \text{ if } Z = B,\ z_t = c_t \text{ if } Z = C.$$

Denote by $\vec{Y}_t^Z = (y_t^Z(1), ..., y_t^Z(m_t{+}1))$ the resulting distribution of seats: $\vec{Y}_t^Z = F(\vec{V}_t^Z)$. The number of seats of party i ($i = 1, ..., m_t$) is given by $y_t^Z(i)$, while the number of seats that would correspond to Z is given by $y_t^Z(m_t + 1)$. These seats are referred to as *ghost seats* because they are not occupied in the parliament.

The set (or number) of parties with parliamentary representation associated with apportionment $\vec{Y}_t^Z$ is denoted by N_t^Z (or n_t^Z). That is, $N_t^Z = \{i \in M_t \mid y_t^Z(i) \neq 0\}$. Let $\vec{W}_t^Z$ denote the corresponding vector of seats: $\vec{W}_t^Z = (w_t^Z(1), ..., w_t^Z(n_t^Z))$ with $w_t^Z(i) = y_t^Z(i)$ for any $i \in N_t^Z$.

The objective is to quantify the effect of replacing $\vec{V}_t^0$ by $\vec{V}_t^Z$ for $Z = A, B, C$ in the parliament. If $y_t^Z(m_t + 1) = 0$ then $y_t^Z(i) = y_t^0(i)$ for $i = 1, ..., m_t$ and $\vec{W}_t^Z = \vec{W}_t^0$ as well as $N_t^Z = N_t^0$. In this case category Z of non votes has no effect. By contrast if $y_t^Z(m_t + 1) \neq 0$ then $\vec{W}_t^Z \neq \vec{W}_t^0$ and the number of seats is reduced: $w_t^Z(i) \leq w_t^0(i)$ for any party with representation in the Parliament ($i \in N_t^Z$). The total number of seats occupied by parties is fewer in $\vec{W}_t^Z$ than in $\vec{W}_t^0$:

$$\sum_{i \in N_t^0} w_t^0(i) = k_t \text{ while } \sum_{i \in N_t^Z} w_t^Z(i) = k_t - y_t^Z(m_t + 1).$$

Moreover $N_t^Z \subseteq N_t^0$ (and thus $n_t^Z \leq n_t^0$). If the inclusion is strict at least one party loses its representation in the Parliament.[5]

If perfect party discipline is assumed the analysis can be taken a step further and decision-making in the parliament can be modeled as weighted majorities among the parties. The weights are the respective number of seats of the different parties and the quota is half the total number of seats (i.e. $k_t/2$). This makes it possible to address the questions of how these alternative apportionments modify the ease with which proposals are adopted, and whether a party will obtain its preferred outcome more easily. Some definitions are required to answer these questions.

A *voting rule* is a well-specified procedure for making decisions among n voters[6]. Let N denote the set of voters. Once a proposal is submitted, voters cast votes. A *vote configuration* is a possible or conceivable result of a vote, that lists the votes cast by all voters. If all non 'yes'-votes are assimilated to 'no'-votes, there are 2^n possible configurations of votes. Each configuration can be represented by the set of 'yes'-voters. We thus use the vote configuration S to refer to the result of a vote where only the voters in S vote 'yes', while those in $N \backslash S$, vote (or are assimilated to) 'no'.

A voting rule is fully specified by the set of vote configurations that would lead to the passing of a proposal. These configurations are referred to as *winning configurations*. In what follows $\mathcal{W}$ denotes the set of winning configurations representing a voting rule. It is assumed that a voting rule satisfies the following requirements: (i) $N \in \mathcal{W}$, (ii) $\emptyset \notin \mathcal{W}$, (iii) If $S \in \mathcal{W}$, then $T \in \mathcal{W}$ for any T containing S, (iv) if $S \in \mathcal{W}$ then $N \backslash S \notin \mathcal{W}$.

[5] These are the $n_t^0 - n_t^A$ smallest parties of N_t^0.

[6] Here the voters are the parties with representation in the parliament.

Symmetric voters are interchangeable in the voting rule. Voters i and j are symmetric in $\mathcal{W}$ if for any vote configuration S such that $i, j \notin S$,

$$S \cup i \in \mathcal{W} \Leftrightarrow S \cup j \in \mathcal{W}.$$

A *vetoer* can prevent the passing of a proposal: if the vote from a vetoer is 'no' then the proposal is rejected. That is, i is a vetoer in $\mathcal{W}$ if

$$i \notin S \Rightarrow S \notin \mathcal{W}.$$

A *null voter* is a voter whose vote never makes a difference. In other words the vote of the other voters determine the outcome irrespective of this voter's vote. That is, i is a null voter in $\mathcal{W}$ if

$$S \in \mathcal{W} \Leftrightarrow S \setminus i \in \mathcal{W}.$$

In a *weighted majority rule* a vector of *weights* $\vec{W} = (w(1), .., w(n))$ is associated with voters, so that the final result is 'yes' if the sum of the weights in favor of the proposal exceeds a given quota Q, with $0 < Q < \sum_{i \in N} w(i)$:

$$\mathcal{W} = \{S \subseteq N : \sum_{i \in S} w(i) > Q\}.$$

Of course if $w(i) = w(j)$ then i and j are symmetric but voters may be symmetric with different weights. In the *unanimity* rule the proposal is accepted if there is a unanimous support:

$$\mathcal{W} = \{N\}.$$

Several features can be evaluated in a voting situation, such as the probability of a proposal being accepted or a voter obtaining his/her preferred outcome. These features require a second input: a probability distribution over the set of vote configurations. Let $p(S)$ denote the probability of S being the vote configuration. For normative purposes we assume here that all vote configurations are equally probable $p(S) = \frac{1}{2^n}$ for any S. For a discussion of this assumption, see Laruelle and Valenciano (2005, 2008).

The probability of a proposal being accepted in rule $\mathcal{W}$ is given by

$$\alpha(\mathcal{W}) = \sum_{S \in \mathcal{W}} \frac{1}{2^n}. \tag{2}$$

Voter i's probability of obtaining the result that voter i voted for in rule $\mathcal{W}$ is denoted by $\Omega(i, \mathcal{W})$. It is given by

$$\Omega(i, \mathcal{W}) = \sum_{\substack{S: \\ i \in S \in \mathcal{W}}} \frac{1}{2^n} + \sum_{\substack{S: \\ i \notin S \notin \mathcal{W}}} \frac{1}{2^n}. \tag{3}$$

We also deal with 'interim' evaluations (i.e., conditional expectations updated with the private information of each voter's own vote). Let $\Omega^+(i, \mathcal{W})$ denote voter i's probability of getting the proposal accepted given that voter i favors the proposal, and $\Omega^-(i, \mathcal{W})$ denote voter i's probability of getting the proposal rejected given that voter i is against the proposal. These conditional probabilities are given by

$$\Omega^+(i, \mathcal{W}) = \sum_{\substack{S: \\ i \in S \in \mathcal{W}}} \frac{1}{2^{n-1}} \text{ and } \Omega^-(i, \mathcal{W}) = \sum_{\substack{S: \\ i \notin S \notin \mathcal{W}}} \frac{1}{2^{n-1}}. \tag{4}$$

Below we use the following notation: $\vec{\Omega}(\mathcal{W}) = (\Omega(1, \mathcal{W}), ..., \Omega(n, \mathcal{W}))$, $\vec{\Omega}^+(\mathcal{W}) = (\Omega^+(1, \mathcal{W}), ..., \Omega^+(n, \mathcal{W}))$ and $\vec{\Omega}^-(\mathcal{W}) = (\Omega^-(1, \mathcal{W}), ..., \Omega^-(n, \mathcal{W}))$. The following properties are worth mentioning (we omit their proof as obvious[7]).

For any rule $\mathcal{W}$,

$$\alpha(\mathcal{W}) \leq \frac{1}{2}, \text{ and } \Omega(i, \mathcal{W}) = \frac{\Omega^+(i, \mathcal{W}) + \Omega^-(i, \mathcal{W})}{2} \text{ for any } i \in N.$$

In the weighted majority $\mathcal{W}$ with $\sum_{i \in N} w(i) = 2Q + 1$ the following equalities hold:

$$\alpha(\mathcal{W}) = \frac{1}{2} \text{ and } \Omega^+(i, \mathcal{W}) = \Omega^-(i, \mathcal{W}) = \Omega(i, \mathcal{W}) \text{ for any } i \in N. \tag{5}$$

If i is a vetoer in $\mathcal{W}$, then

$$\Omega^-(i, \mathcal{W}) = 1.$$

If i and j are symmetric in $\mathcal{W}$, then

$$\Omega^+(i, \mathcal{W}) = \Omega^+(j, \mathcal{W}), \; \Omega^-(i, \mathcal{W}) = \Omega^-(j, \mathcal{W}), \; \Omega(i, \mathcal{W}) = \Omega(j, \mathcal{W}). \tag{6}$$

If i is a null voter in $\mathcal{W}$, then

$$\Omega^+(i, \mathcal{W}) = \alpha(\mathcal{W}), \; \Omega^-(i, \mathcal{W}) = 1 - \alpha(\mathcal{W}), \text{ and } \Omega(i, \mathcal{W}) = \frac{1}{2}. \tag{7}$$

If $\mathcal{W} = \{N\}$, then

$$\alpha(\mathcal{W}) = \frac{1}{2^n}, \; \Omega^+(i, \mathcal{W}) = \frac{1}{2^{n-1}}, \; \Omega^-(i, \mathcal{W}) = 1, \text{ and } \Omega(i, \mathcal{W}) = \frac{1 + 2^{n-1}}{2^n} \text{ for any } i \in N.$$

On the basis of the vectors of distribution of seats $\vec{W}_t^0$, $\vec{W}_t^A$, $\vec{W}_t^B$, and $\vec{W}_t^C$ it is possible to derive the weighted majorities $\mathcal{W}_t^0$, $\mathcal{W}_t^A$, $\mathcal{W}_t^B$, and $\mathcal{W}_t^C$, all of them with an identical quota of half the number of seats in the parliament: $Q_t = k_t/2$. We then compare $\mathcal{W}_t^0$ with $\mathcal{W}_t^Z$ for $Z = A, B, C$ by computing (2), (3) and (4) for $\mathcal{W}_t^0$ and $\mathcal{W}_t^Z$. In the next section we study in detail the case of the Basque Parliament.

[7] To obtain (5) note that in such a weighted majority $S \in \mathcal{W}$ if and only if $N \backslash S \notin \mathcal{W}$ on the one hand and $i \in S$ if and only if $i \notin N \backslash S$ on the other hand. Property (7) is a direct corollary of Proposition 2 in Laruelle et al. (2006).

3 Application to the Basque Parliament

We focus on the Parliament of the Basque Autonomous Community, one of the 17 regions in Spain. The party system is described in the Appendix. The website of the Department of Security of the Basque Government[8] provides all electoral results since the first elections in 1977, including the number of abstentions, null and blank votes.

Ten elections to the Basque Parliament have been held between 1977 and 2015: $t = 1980$, 1984, 1986, 1990, 1994, 1998, 2001, 2005, 2009 and 2012. The number of seats in the Basque Parliament (k_t) has been 75 since 1984, though it was 60 in the 1980 election. There are three electoral districts corresponding to the three provinces: Biscay, Gipuzcoa and Alava. In each district one third[9] of the total number of seats is allocated according to the D'Hondt method with a threshold of 3% in all electoral years except 1986 and 1998 when it was 5%.

By contrast with other European countries the Spanish party system is characterized by the existence of many non state-wide parties (Pallarés et al., 1997). This explains the large number of parties that compete in elections (m_t), as shown in Table 1: The figure varies between eight and nineteen, with an average of twelve.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
m_t	16	10	13	19	8	10	10	12	14	19

Table 1: Number of parties competing for seats in the Basque Parliament

For each electoral year the actual apportionment ($\vec{Y}_t^0$) in the Parliament is obtained by adding up the apportionments in the different districts. The set of parties with representation (N_t^0) is obtained by restricting the set of parties competing to those with at least one seat. It is given by (17) in the Appendix.

t	e_t	a_t	b_t	c_t	n_t^0	$v_t(1)$	$v_t(n_t^0)$
1980	1,554,527	625,476	3,570	9,206	7	349,102	36,845
1984	1,584,540	499,236	5,029	6,247	5	451,178	85,671
1986	1,660,143	504,328	5,003	6,737	7	252,233	40,445
1990	1,687,936	658,479	7,580	5,163	7	289,701	14,351
1994	1,749,250	705,165	18,080	6,184	7	304,346	27,797
1998	1,821,608	546,600	17,641	6,802	7	350,322	15,738
2001	1,813,356	381,360	11,508	6,219	5	604,222	78,862
2005	1,799,500	575,866	9,001	4,035	6	468,117	28,180
2009	1,776,059	627,362	11,562	100,939	7	399,600	22,233
2012	1,775,351	639,783	14,640	9,168	5	384,766	21,539

Table 2: Electoral results in the Basque Parliament

[8] http://www.euskadi.net/

[9] Each province elects the same number of representatives in spite of an average ratio of populations between the three provinces of (4.2: 2.5: 1).

Table 2 summarizes the results of the different elections in terms of electorate (e_t), abstention (a_t), blank votes (b_t) and null votes (c_t), as well as the number of parties with representation in parliament (n_t^0). The last two columns show the number of votes obtained by the party with the largest number of seats ($v_t(1)$) and the smallest party with a parliamentary representation ($v_t(n_t^0)$). As can be seen from Table 2, the number of abstainers is always substantial, and averages 33.6% of the electorate. The trend was decreasing in the eighties, peaking in 1980 (40.23%). Then it was increasing in the early nineties, and decreasing in the late nineties, reaching its lowest level in 2001 (21.03%). Since then it has increased again. The number of abstainers is larger than the number of votes for the largest party in all cases except 2001. The number of seats allocated to abstention can thus be expected to be large if these votes are taken into account. The percentage of blank votes has always been low, between 0.22% and 1% of the total electorate. The number of blank votes is smaller than the number of votes of the smallest party with a parliamentary representation in all cases except 1998, when it obtained more votes than the smallest party, which obtained two seats. The percentage of null votes is low at between 0.2% and 0.6% of the electorate, with the exception of 2009, when it obtained 100,939 votes and accounted for 5.7% of the total electorate. The number of parties with parliamentary representation (n_t^0) varies between five and seven, while the actual distribution of seats ($\vec{W}_t^0$) is the following:

$$\begin{array}{l} \vec{W}^0_{1980} = (25, \; 11, \; 9, \; 6, \; 6, \; 2, \; 1) \\ \vec{W}^0_{1984} = (32, \; 19, \; 11, \; 7, \; 6) \\ \vec{W}^0_{1986} = (19, \; 17, \; 13, \; 13, \; 9, \; 2, \; 2) \\ \vec{W}^0_{1990} = (22, \; 16, \; 13, \; 9, \; 6, \; 6, \; 3) \\ \vec{W}^0_{1994} = (22, \; 12, \; 11, \; 11, \; 8, \; 6, \; 5) \\ \vec{W}^0_{1998} = (21, \; 16, \; 14, \; 14, \; 6, \; 2, \; 2) \\ \vec{W}^0_{2001} = (33, \; 19, \; 13, \; 7, \; 3) \\ \vec{W}^0_{2005} = (29, \; 18, \; 15, \; 9, \; 3, \; 1) \\ \vec{W}^0_{2009} = (30, \; 25, \; 13, \; 4, \; 1, \; 1, \; 1) \\ \vec{W}^0_{2012} = (27, \; 21, \; 16, \; 10, \; 1) \end{array} \tag{8}$$

Whenever there is a single district the apportionment method satisfies the monotonicity condition (1). This property may be violated when there are several districts. This happens in three of the ten electoral years. In 1986 the most voted party (271,208 votes) obtained fewer seats (17) than did the second most voted party (19 seats for 252,233 votes), because the most voted party got more votes in one district, where it got two more seats, while in the other two districts the second most voted party got more votes and two more seats in both. In 1990 the 14,351 votes that went to the smallest party with representation in parliament were mainly concentrated in one district, where it obtained three seats, while no seats were allocated to a party that obtained 14,440 votes. In 2012 the largest party without representation in the parliament obtained 30,318 votes while the smallest party with representation obtained one seat with 21,539 votes.

For each electoral year t the vector of weight $\vec{W}_t^0$ and the quota Q_t (with $Q_{1980} = 30$ and $Q_t = 37.5$ for $t \neq 1980$) define the actual voting rule among the parties, which we denote by $\mathcal{W}_t^0$. Analyzing them we can observe classical properties in weighted majorities: (i) many parties appear to be symmetric in spite of different numbers of seats; and (ii) some parties are null voters in spite of having a strictly positive number of seats.

(i) In $\mathcal{W}_{1980}^0$ the second, third, fourth and fifth largest parties are symmetric with 11, 9, 6, and 6 seats, respectively. The same goes for the two smallest parties with 2 and 1 seats, respectively. In $\mathcal{W}_{1984}^0$ all parties except the largest are symmetric with 19, 11, 7, and 6 seats, respectively. In $\mathcal{W}_{1986}^0$ the two largest parties (with 19 and 17 seats, respectively), and the third, fourth and fifth parties (with 13, 13, and 9 seats, respectively) are symmetric. In $\mathcal{W}_{1990}^0$ the three smallest parties are symmetric (with 6, 6, and 3 seats, respectively). In $\mathcal{W}_{1994}^0$ the second, third, and fourth largest parties (with 12, 11, and 11 seats, respectively), and the three smallest parties (with 8, 6, and 5 seats, respectively) are symmetric. In $\mathcal{W}_{2001}^0$ the second, third and fourth largest parties (with 19, 13 and 7 seats, respectively) are symmetric. In $\mathcal{W}_{2005}^0$ the second, third and fourth parties are symmetric (with 18, 15 and 9 seats, respectively), as are the two smallest parties (with 3 and 1 seats, respectively). In $\mathcal{W}_{2009}^0$ the three largest parties are symmetric (with 30, 25, and 13 seats, respectively), and so are the four smallest parties (with 4, 1, 1, and 1 seats, respectively). In $\mathcal{W}_{2012}^0$ the second and third largest are symmetric (with 21 and 16 seats, respectively), as are the two smallest parties (with 10 and 1 seats, respectively).

(ii) Existence of null voters in spite of strictly positive weights. In $\mathcal{W}_{1980}^0$ the two smallest parties are null voters (with 2 and 1 seats, respectively). In $\mathcal{W}_{2001}^0$ the smallest party is a null voter (with 3 seats). In $\mathcal{W}_{2005}^0$ the two smallest parties are null voters (with 3 and 1 seats, respectively). In $\mathcal{W}_{2009}^0$ the four smallest parties are null voters (with 4, 1, 1, and 1 seats, respectively).

The difference in seats between symmetric parties can be large (as many as 17 seats in $\mathcal{W}_{2009}^0$). A party with 4 seats can be a null voter (in $\mathcal{W}_{2009}^0$). Note however that in $\mathcal{W}_{2012}^0$ the smallest party -with 1 seat- is not a null voter.

We continue the analysis of $\mathcal{W}_t^0$ by computing the normative probabilities. First note that as a direct application of (5),[10] for any t

$$\alpha(\mathcal{W}_t^0) = \frac{1}{2} \text{ and } \Omega(i, \mathcal{W}_t^0) = \Omega^+(i, \mathcal{W}_t^0) = \Omega^-(i, \mathcal{W}_t^0) \text{ for any } i \in N_t^0. \tag{9}$$

[10] Although the total number of seats is even for $t = 1980$ there is no configuration of votes such that the total weight is exactly 30 (and thus $S \in \mathcal{W}$ if and only if $N \backslash S \notin \mathcal{W}$).

Secondly, symmetric parties have equal probabilities by (6), and equalities (7) hold for null voters. As equalities (9) hold only the probabilities of the parties getting their preferred outcomes are given:

$$\begin{array}{l}
\vec{\Omega}(\mathcal{W}^0_{1980}) = (0.94,\ \ 0.56,\ \ 0.56,\ \ 0.56,\ \ 0.56,\ \ 0.50,\ \ 0.50) \\
\vec{\Omega}(\mathcal{W}^0_{1984}) = (0.94,\ \ 0.56,\ \ 0.56,\ \ 0.56,\ \ 0.56) \\
\vec{\Omega}(\mathcal{W}^0_{1986}) = (0.73,\ \ 0.73,\ \ 0.64,\ \ 0.64,\ \ 0.64,\ \ 0.52,\ \ 0.52) \\
\vec{\Omega}(\mathcal{W}^0_{1990}) = (0.80,\ \ 0.70,\ \ 0.67,\ \ 0.58,\ \ 0.55,\ \ 0.55,\ \ 0.55) \\
\vec{\Omega}(\mathcal{W}^0_{1994}) = (0.84,\ \ 0.63,\ \ 0.63,\ \ 0.63,\ \ 0.56,\ \ 0.56,\ \ 0.56) \\
\vec{\Omega}(\mathcal{W}^0_{1998}) = (0.77,\ \ 0.70,\ \ 0.64,\ \ 0.64,\ \ 0.61,\ \ 0.55,\ \ 0.55) \\
\vec{\Omega}(\mathcal{W}^0_{2001}) = (0.88,\ \ 0.63,\ \ 0.63,\ \ 0.63,\ \ 0.50) \\
\vec{\Omega}(\mathcal{W}^0_{2005}) = (0.88,\ \ 0.63,\ \ 0.63,\ \ 0.63,\ \ 0.50,\ \ 0.50) \\
\vec{\Omega}(\mathcal{W}^0_{2009}) = (0.75,\ \ 0.75,\ \ 0.75,\ \ 0.50,\ \ 0.50,\ \ 0.50,\ \ 0.50) \\
\vec{\Omega}(\mathcal{W}^0_{2012}) = (0.81,\ \ 0.69,\ \ 0.69,\ \ 0.56,\ \ 0.56)
\end{array} \tag{10}$$

The probability of the largest party of obtaining its preferred outcome is between 0.73 and 0.94, with an average of 0.83 over the ten electoral years. For the smallest party the probability varies between 0.5 (when it is a null voter) and 0.56, with an average of 0.53.

The same apportionment method is then applied to the vectors of votes adding a fictitious party whose number of votes is equal to one category of the non votes to obtain $\vec{Y}_t^Z$ for $Z = A, B, C$ and $t = 1980, ..., 2012$. As shown below, blank seats would have no effect, null seats would have an effect in 2009, and abstention would always have an effect.

Year	Biscay	Gipuzcoa	Alava
1980	21,069	12,380	4,521
1984	21,826	13,035	4,498
1986	33,733	19,688	7,408
1990	19,291	11,410	4,264
1994	20,184	11,522	4,980
1998	36,462	21,612	9,030
2001	27,996	16,954	6,971
2005	22,970	22,970	6,174
2009	20,145	10,982	5,281
2012	20,928	12,717	5,060

Table 3: Minimum number of votes to achieve one ghost seat

For any t, $y_t^B(m_t + 1) = 0$: the number of blank votes is never sufficient to obtain a seat. Recall (from Table 2) that the number of blank votes is always lower than the number of votes of the smallest party with representation in the parliament (except in 1998). In 1998 blank votes would get no seats because they are relatively equally distributed in the three districts, while the smallest party with representation in the parliament concentrates its votes in one district, where it manages to obtain two seats. This raises the question of how many votes would be required to obtain a ghost seat. Table 3 answers this question, province by province. The table shows that in 1990 just 4,264 votes would suffice to obtain a ghost seat as long as they were concentrated in the smallest province, Alava.

With the exception of 2009, the number of null votes is never sufficient to obtain a seat either. As observed in Table 2 the number of null votes is always lower than the number of votes of the smallest party with representation in the parliament (except in 2009). We obtain $y_t^C(m_t+1) = 0$ for $t \neq 2009$ and

$$y_{2009}^C(m_t+1) = 7, \quad n_{2009}^C = 6, \quad \vec{W}_{2009}^C = (28, \quad 23, \quad 11, \quad 4, \quad 1, \quad 1). \tag{11}$$

If the null votes were treated as a party they would obtain seven seats. Recalling that $n_{2009}^0 = 7$, $n_{2009}^C = 6$ means that the smallest party with representation in the actual parliament would lose its representation. Comparing (8) with (11) it can be seen that $w_{2009}^C(i) < w_{2009}^0(i)$ holds for any $i = 1, 2, 3$ while $w_{2009}^C(i) = w_{2009}^0(i)$ for $i = 4, 5, 6$. The three largest parties lose seats, the smallest party loses its representation and the others keep the same number of seats. Rule $\mathcal{W}_{2009}^C$ is the weighted majority with weights (11) and quota Q_t. This rule exhibits the following properties: (i) the second and third largest parties are symmetric (with 23 and 11 seats, respectively), (ii) the two smallest parties (with 1 seat) are null voters. Computing (2), (3) and (4) for $\mathcal{W}_{2009}^C$ respectively gives the following:

$$\begin{array}{rl} \alpha(\mathcal{W}_{2009}^C) = & 0.44, \\ \vec{\Omega}^+(\mathcal{W}_{2009}^C) = & (0.75, \quad 0.63, \quad 0.63, \quad 0.5, \quad 0.44, \quad 0.44) \\ \vec{\Omega}^-(\mathcal{W}_{2009}^C) = & (0.88, \quad 0.75, \quad 0.75, \quad 0.63, \quad 0.56, \quad 0.56) \\ \vec{\Omega}(\mathcal{W}_{2009}^C) = & (0.81, \quad 0.69, \quad 0.69, \quad 0.56, \quad 0.5, \quad 0.5) \end{array} \tag{12}$$

Compare (12) with (9) and (10) it can be seen that the following inequalities hold:

$$\alpha(\mathcal{W}_{2009}^C) < \alpha(\mathcal{W}_{2009}^0), \text{ and } \Omega^+(i, \mathcal{W}_{2009}^C) \leq \Omega^+(i, \mathcal{W}_{2009}^0) = \Omega^-(i, \mathcal{W}_{2009}^0) \leq \Omega^-(i, \mathcal{W}_{2009}^C)$$

for any $i \in N_{2009}^C$. It is more difficult to adopt a proposal with $\mathcal{W}_{2009}^C$ than with $\mathcal{W}_{2009}^0$. Any party is more likely to obtain its preferred outcome in $\mathcal{W}_{2009}^C$ if it is against the proposal, but less likely to do so if it is in favor of the proposal. The overall effect is not identical for all parties as $\Omega(i, \mathcal{W}_{2009}^C) > \Omega(i, \mathcal{W}_{2009}^0)$ for $i = 1, 4$ while $\Omega(i, \mathcal{W}_{2009}^C) < \Omega(i, \mathcal{W}_{2009}^0)$ holds for $i = 2, 3, 5, 6$.

By contrast abstention would have a substantial effect on decision-making if it counted as a party in the apportionment method. As shown in Table 2 the number of abstainers is larger than the number of votes received by the largest party in all years except 2001. Table 4 gives the number of seats that would correspond to the abstention if it were treated as a party and the number of parties with representation in parliament for each electoral year.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
$y_t^A(m_t+1)$	29	26	26	33	34	26	16	26	32	32
n_t^A	5	5	6	7	7	6	5	5	4	4

Table 4: Number of ghost seats corresponding to abstention and number of parties.

The number of ghost seats is between 16 and 34 with an average of 28 seats out of 75 between 1984 and 2012. It always accounts for more than 20% of the total number of seats in the parliament, and as much as 48% in 1980 (29 seats out of 60). A comparison of Tables 2 and 4 shows that in most electoral years at least one party would lose its representation in the parliament.

The distribution of seats among the parties is modified as follows:

$$\begin{array}{l}
\vec{W}^A_{1980} = (14, \quad 6, \quad 4, \quad 4, \quad 3) \\
\vec{W}^A_{1984} = (22, \quad 12, \quad 7, \quad 5, \quad 3) \\
\vec{W}^A_{1986} = (12, \quad 12, \quad 9, \quad 9, \quad 6, \quad 1) \\
\vec{W}^A_{1990} = (13, \quad 9, \quad 8, \quad 5, \quad 3, \quad 3, \quad 1) \\
\vec{W}^A_{1994} = (13, \quad 6, \quad 6, \quad 6, \quad 4, \quad 3, \quad 3) \\
\vec{W}^A_{1998} = (14, \quad 13, \quad 10, \quad 9, \quad 2, \quad 1) \\
\vec{W}^A_{2001} = (25, \quad 15, \quad 11, \quad 5, \quad 3) \\
\vec{W}^A_{2005} = (20, \quad 12, \quad 10, \quad 5, \quad 2) \\
\vec{W}^A_{2009} = (19, \quad 16, \quad 7, \quad 1) \\
\vec{W}^A_{2012} = (16, \quad 12, \quad 9, \quad 6)
\end{array} \tag{13}$$

Comparing (8) and (13) it can be observed that the seats lost by the largest parties are in general lower in percentage terms than those lost by the smallest parties. For instance in 1980, the two largest parties would lose less than 50% of their seats (from 25 seats to 14 or from 11 to 6) while the two smallest parties would lose their representation and the third smallest party would lose exactly 50% (from 6 seats to 3).

In electoral year t, rule $\mathcal{W}^A_t$ is the weighted majority with weights $\vec{W}^A_t$ given in (13) and quota Q_t. Analyzing the voting rules again leads the classical results of (i) symmetric parties; and (ii) null voters. The new property observed in $\mathcal{W}^A_t$ is that (iii) the largest party is a vetoer in all electoral years.

(i) In $\mathcal{W}^A_{1980}$ all parties are symmetric (with 14, 6, 4, 4 and 3 seats, respectively). In $\mathcal{W}^A_{1984}$ the two largest parties (with 22 and 12 seats, respectively) and the third and fourth largest parties (with 7 and 5 seats, respectively) are symmetric. In $\mathcal{W}^A_{1986}$ the third, fourth and fifth largest parties are symmetric (with 9, 9 and 6 seats, respectively). In $\mathcal{W}^A_{1990}$ the four largest parties (with 13, 9, 8 and 5 seats, respectively) are symmetric. In $\mathcal{W}^A_{1994}$ the five largest parties (with 13, 6, 6, 6 and 4 seats, respectively) are symmetric. In $\mathcal{W}^A_{1998}$ the two largest parties are symmetric (with 14 and 13 seats, respectively). The same goes for the fourth and fifth largest parties (with 9 and 2 seats, respectively). In $\mathcal{W}^A_{2001}$ the two smallest parties are symmetric (with 5 and 3 seats, respectively). In $\mathcal{W}^A_{2005}$ the two largest parties (with 20 and 12 seats, respectively) and the two smallest parties (with 5 and 2 seats, respectively) are symmetric. In $\mathcal{W}^A_{2009}$ the three largest parties are symmetric (with 19, 16 and 7 seats, respectively). In $\mathcal{W}^A_{2012}$ all parties are symmetric (with 16, 12, 9 and 6 seats, respectively).

(ii) In $\mathcal{W}^A_{1984}$, $\mathcal{W}^A_{1986}$, $\mathcal{W}^A_{1990}$, $\mathcal{W}^A_{2001}$ and $\mathcal{W}^A_{2009}$ the smallest party is a null voter (with 3, 1, 1, 3 or 1 seats, respectively).

(iii) In $\mathcal{W}^A_{2001}$ the largest party is a vetoer. In $\mathcal{W}^A_{1984}$, $\mathcal{W}^A_{1986}$, $\mathcal{W}^A_{1998}$ and $\mathcal{W}^A_{2005}$ the two largest parties are vetoers. In $\mathcal{W}^A_{2009}$ the three largest parties are vetoers. In $\mathcal{W}^A_{1990}$ the four largest parties are vetoers. In $\mathcal{W}^A_{1994}$ the five largest parties are vetoers. Note that $\mathcal{W}^A_{1980}$ and $\mathcal{W}^A_{2012}$ are unanimities: all parties are vetoers.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
$\alpha(\mathcal{W}^A_t)$	0.03	0.19	0.13	0.05	0.02	0.14	0.34	0.16	0.13	0.06

Table 5: probabilities of a proposal being adopted in $\mathcal{W}^A_t$.

Table 5 presents the results of computing (2) for $\mathcal{W}^A_t$. As can be observed from the table, the probability of a proposal being adopted varies between 0.02 and 0.34. It is very low, nearly always below 0.2. This represents an important decrease compared to $\alpha(\mathcal{W}^0_t) = 0.5$. The average over the ten electoral years is 0.13. Note that the probability of a proposal being adopted follows the opposite path to the abstention: when the level of abstention decreases the probability a proposal being adopted in $\mathcal{W}^A_t$ increases.

Computing (4) for $\mathcal{W}^A_t$ gives the following:

$$\begin{aligned}
\vec{\Omega}^+(\mathcal{W}^A_{1980}) &= (0.07,\ 0.07,\ 0.07,\ 0.07,\ 0.07)\\
\vec{\Omega}^+(\mathcal{W}^A_{1984}) &= (0.38,\ 0.38,\ 0.25,\ 0.25,\ 0.19)\\
\vec{\Omega}^+(\mathcal{W}^A_{1986}) &= (0.25,\ 0.25,\ 0.19,\ 0.19,\ 0.19,\ 0.13)\\
\vec{\Omega}^+(\mathcal{W}^A_{1990}) &= (0.09,\ 0.09,\ 0.09,\ 0.09,\ 0.06,\ 0.06,\ 0.05)\\
\vec{\Omega}^+(\mathcal{W}^A_{1994}) &= (0.05,\ 0.05,\ 0.05,\ 0.05,\ 0.05,\ 0.03,\ 0.03)\\
\vec{\Omega}^+(\mathcal{W}^A_{1998}) &= (0.28,\ 0.28,\ 0.19,\ 0.22,\ 0.19,\ 0.16)\\
\vec{\Omega}^+(\mathcal{W}^A_{2001}) &= (0.69,\ 0.5,\ 0.44,\ 0.38,\ 0.38)\\
\vec{\Omega}^+(\mathcal{W}^A_{2005}) &= (0.31,\ 0.31,\ 0.25,\ 0.19,\ 0.19)\\
\vec{\Omega}^+(\mathcal{W}^A_{2009}) &= (0.25,\ 0.25,\ 0.25,\ 0.13)\\
\vec{\Omega}^+(\mathcal{W}^A_{2012}) &= (0.13,\ 0.13,\ 0.13,\ 0.13)
\end{aligned} \tag{14}$$

The probabilities are very small in $\mathcal{W}^A_{1980}$, $\mathcal{W}^A_{1990}$, $\mathcal{W}^A_{1994}$, and $\mathcal{W}^A_{2012}$: the values do not reach 0.1 for the largest party. The average probability is 0.25 for the largest party, and is above 0.5 only in $\mathcal{W}^A_{2001}$. For the other parties the values are always below (or in one case equal to) 0.5.

Computing (4) for $\mathcal{W}^A_t$ gives the following:

$$\begin{array}{lllllll}
\vec{\Omega}^-(\mathcal{W}^A_{1980}) = (1, & 1, & 1, & 1, & 1) & & \\
\vec{\Omega}^-(\mathcal{W}^A_{1984}) = (1, & 1, & 0.88, & 0.88, & 0.81) & & \\
\vec{\Omega}^-(\mathcal{W}^A_{1986}) = (1, & 1, & 0.94, & 0.94, & 0.94 & 0.88) & \\
\vec{\Omega}^-(\mathcal{W}^A_{1990}) = (1, & 1, & 1, & 1, & 0.97, & 0.97, & 0.95) \\
\vec{\Omega}^-(\mathcal{W}^A_{1994}) = (1, & 1, & 1, & 1, & 1, & 0.98, & 0.98) \\
\vec{\Omega}^-(\mathcal{W}^A_{1998}) = (1, & 1, & 0.91, & 0.94, & 0.91, & 0.88) & \\
\vec{\Omega}^-(\mathcal{W}^A_{2001}) = (1, & 0.81, & 0.75, & 0.69, & 0.69) & & \\
\vec{\Omega}^-(\mathcal{W}^A_{2005}) = (1, & 1, & 0.94, & 0.88, & 0.88) & & \\
\vec{\Omega}^-(\mathcal{W}^A_{2009}) = (1, & 1, & 1, & 0.88) & & & \\
\vec{\Omega}^-(\mathcal{W}^A_{2012}) = (1, & 1, & 1, & 1) & & &
\end{array} \tag{15}$$

The probability of a party obtaining its preferred outcome if it is against a proposal is very high: it is above 0.8 in all cases for all parties (with the exception of $\mathcal{W}^A_{2001}$ where the values are between 0.69 and 0.75 for the three smallest parties). There are many vetoers (whose probability is 1).

Computing (3) for $\mathcal{W}^A_t$ gives the following:

$$\begin{array}{lllllll}
\vec{\Omega}(\mathcal{W}^A_{1980}) = (0.53, & 0.53, & 0.53, & 0.53, & 0.53) & & \\
\vec{\Omega}(\mathcal{W}^A_{1984}) = (0.69, & 0.69, & 0.56, & 0.56, & 0.5) & & \\
\vec{\Omega}(\mathcal{W}^A_{1986}) = (0.63, & 0.63, & 0.56, & 0.56, & 0.56, & 0.5) & \\
\vec{\Omega}(\mathcal{W}^A_{1990}) = (0.55, & 0.55, & 0.55, & 0.55, & 0.52, & 0.52, & 0.5) \\
\vec{\Omega}(\mathcal{W}^A_{1994}) = (0.52, & 0.52, & 0.52, & 0.52, & 0.52, & 0.51, & 0.51) \\
\vec{\Omega}(\mathcal{W}^A_{1998}) = (0.64, & 0.64, & 0.55, & 0.58, & 0.55, & 0.52) & \\
\vec{\Omega}(\mathcal{W}^A_{2001}) = (0.84, & 0.65, & 0.59, & 0.53, & 0.53) & & \\
\vec{\Omega}(\mathcal{W}^A_{2005}) = (0.66, & 0.66, & 0.59, & 0.53, & 0.53) & & \\
\vec{\Omega}(\mathcal{W}^A_{2009}) = (0.63, & 0.63, & 0.63, & 0.5) & & & \\
\vec{\Omega}(\mathcal{W}^A_{2012}) = (0.56, & 0.56, & 0.56, & 0.56) & & &
\end{array} \tag{16}$$

Comparing (10), (14), (15) and (16) it can be observed that

$$\Omega^+(i, \mathcal{W}^A_t) < \Omega^+(i, \mathcal{W}^0_t) = \Omega^-(i, \mathcal{W}^0_t) < \Omega^-(i, \mathcal{W}^A_t) \text{ for any } i \in N^A_t.$$

Note that the difference between $\Omega^+_i(\mathcal{W}^A_t)$ and $\Omega^+_i(\mathcal{W}^0_t)$ and between $\Omega^-_i(\mathcal{W}^0_t)$ and $\Omega^-_i(\mathcal{W}^A_t)$ are large. If abstention were treated as a party in the apportionment method the probability of a party obtaining its preferred outcome if it favors (rejects) the proposal would decrease (increase) substantially in the parliament. Comparing (10) and (16) it can be observed that in general $\Omega(i, \mathcal{W}^A_t) \leq \Omega(i, \mathcal{W}^0_t)$ (the three exceptions being $i = 2$ for $t = 1984$ and $i = 5$ for $t = 2001, 2005$). For any party rule $\mathcal{W}^0_t$ generally gives a higher probability of getting its preferred outcomes than rule $\mathcal{W}_t^A$, so it should be preferred (unless the party attaches the greatest importance to the possibility of blocking proposals).

4 Discussion

The potential impact of non voters is usually measured by simulating complete turnout. Lutz and Marsh (2007) review the studies on the impact of turnout on electoral outcomes. They conclude that turnout does not matter a great deal, no matter what method, dataset or period of time the authors consider. Kohler (2011) studies the effect on the formation of governments in Germany from 1949 to 2009. Non voters are assumed to behave like voters with the same social characteristics. He arrives at a similar conclusion: non voters would most likely not have had a big influence on government formation in Germany. Here we depart from the above mentioned assumption regarding non voters' preferences. We basically assume that non voters choose not to vote to express dissatisfaction and factor in an institutional change that permits them to be represented them via vacant seats. The potential impact of non voters would then be substantial. Our results supplement those of previous studies, as some non voters are not interested in politics while others are dissatisfied with the political system, as illustrated by the recent creation of new parties in Italy, Greece and Spain.

The institutional change suggested in this paper brings some insight to the discussion on compulsory voting. Lijpart (1997) supports compulsory voting as it would boost participation and discusses two arguments against it. First, if it forces people with little interest in politics to the polls it may also serve as an incentive to become more informed. Second, although individual freedom is reduced, there is no duty to cast a valid vote. Nevertheless only valid votes really count in the apportionment method. This paper suggests that additional options could be offered to voters. For instance on top of being allowed to vote for candidates, voters could cast an indifference vote or a vote of dissatisfaction. Votes of dissatisfaction could be computed as a party in the apportionment method and the corresponding seats would be left vacant, while indifferent votes would be considered in the same way as blank votes currently are. These additional options would allow dissatisfied citizens to be "represented" in the Parliament by ghost seats. Lijpart (1997) mentions the opposition of conservative parties as an especially big obstacle to the adoption of compulsory voting. The opposition of parties to the introduction of ghost seats may even be stronger.

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5 Appendix: Party system in the Basque Parliament

The parties with representation in Parliament are the following:

$$\begin{aligned}
N^0_{1980} &= \{EAJ\text{-}PNV, HB, PSE\text{-}PSOE, UCD, EE, AP, PCE\text{-}EPK\} \\
N^0_{1984} &= \{EAJ\text{-}PNV, PSE\text{-}PSOE, HB, AP\text{-}PDP\text{-}UL, EE\} \\
N^0_{1986} &= \{PSE\text{-}PSOE, EAJ\text{-}PNV, HB, EA, EE, CDS, AP\text{-}PL\} \\
N^0_{1990} &= \{EAJ\text{-}PNV, PSE\text{-}PSOE, HB, EA, PP, EE, U.AL\} \\
N^0_{1994} &= \{EAJ\text{-}PNV, PSE\text{-}EE/PSOE, HB, PP, EA, EB\text{-}B, U.AL\} \\
N^0_{1998} &= \{EAJ\text{-}PNV, PP, EH, PSE\text{-}EE/PSOE, EA, U.AL, EB\text{-}B\} \\
N^0_{2001} &= \{EAJ\text{-}PNV/EA, PP, PSE\text{-}EE/PSOE, EH, EB\text{-}B\} \\
N^0_{2005} &= \{EAJ\text{-}PNV/EA, PSE\text{-}EE/PSOE, PP, PCTV\text{-}EHAK, EB\text{-}B, Aralar\} \\
N^0_{2009} &= \{EAJ\text{-}PNV, PSE\text{-}EE/PSOE, PP, Aralar, EA, UPyD, EB\text{-}B\} \\
N^0_{2012} &= \{EAJ\text{-}PNV, EH\text{-}Bildu, PSE\text{-}EE/PSOE, PP, UPyD\}. \qquad (17)
\end{aligned}$$

They are ordered by their number of seats. Two of the four largest parties are non state-wide parties, and the other two are state-wide parties. The political projects of the non state-wide parties concern the concept of nation and identity. The largest party in terms of votes is the Basque Nationalist Party (EAJ-PNV), a (non state-wide) right-wing nationalist party. The other nationalist party represents what is known as the Patriotic Left ("Izquierda Abertzale"). It has competed in elections under different names and sometimes in coalitions (HB from 1980 till 1994, EH in 1998 and 2001, Batasuna from 2001 till 2011, and EH-Bildu since 2012). Note that Batasuna was outlawed in 2003 for its connection with the ETA terrorist group. In the 2005 elections Batasuna's supporters mainly voted for the Communist Party of the Basque Lands (PCTV-EHAK). See Pallarés et al. (2007). After this last party was outlawed in 2008 Batasuna called on its supporters to cast a null vote in the 2009 election (and as a result null votes accounted for 5.7% of the electorate). The other two main parties are state-wide parties divided along the left-right cleavage: the right-wing Popular Party (PP, formally AP, AP-PDP-UL, or AP-PL) and the left-wing Socialist Party (PSE-PSOE, or currently PSE-EE/PSOE after a merger with the party EE).

Apart from these four main parties, three others have obtained representation in at least half the elections considered. The Basque Solidarity Party (EA) emerged from a split in the Basque Nationalist Party. It has had parliamentary representation since 1986, alone or in coalition (in 2001 and 2005 with EAJ-PNV, and in 2012 within the EH-Bildu coalition). The Basque Left (EE) was present in the first four legislatures before it merged with the Socialist Party. The United Left and Green Party (EB-B) is a Communist Party which is a partner of the state-wide United Left party (IU). It obtained seats in the five legislatures between 1994 and 2009.

Three parties obtained seats in two or three legislatures. Aralar is a party that was formed in a split from Batasuna when the latter was banned: it stood at the 2005 and 2009 elections before joining the EH-Bildu coalition for the 2012 elections. When the Popular Party was founded in 1989 some members did not agree with its program on the territorial division of the Basque Country and formed the Alavesan Unity Party (UAL). This party obtained seats in three consecutive legislatures (from 1990 till 1998) and was dissolved in 2005. The party of Union, Progress and Democracy (UPyD) was founded by a former socialist leader in 2007 as an anti-nationalist centrist party.

Finally there are other parties that have obtained parliamentary representation in isolated elections: the state-wide centrist parties Union of the Democratic Centre (UCD) and Social Democratic Centre (CDS) (respectively present in the 1980 and 1986 elections), and the Communist Party (PCE-EPK in the 1980 elections). For more details, see Leonisio (2012), Strijbis and Leonisio (2012) or Gomez and Cabeza (2013).